



# Monday 10 June 2013 – Morning

## AS GCE MATHEMATICS

4725/01 Further Pure Mathematics 1

### QUESTION PAPER

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4725/01
- List of Formulae (MF1)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 The complex number  $3 + ai$ , where  $a$  is real, is denoted by  $z$ . Given that  $\arg z = \frac{1}{6}\pi$ , find the value of  $a$  and hence find  $|z|$  and  $z^* - 3$ . [6]

2 The matrices  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are given by  $\mathbf{A} = \begin{pmatrix} 5 & 1 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} 2 & -5 \end{pmatrix}$  and  $\mathbf{C} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ .  
(i) Find  $3\mathbf{A} - 4\mathbf{B}$ . [2]

(ii) Find  $\mathbf{CB}$ . Determine whether  $\mathbf{CB}$  is singular or non-singular, giving a reason for your answer. [5]

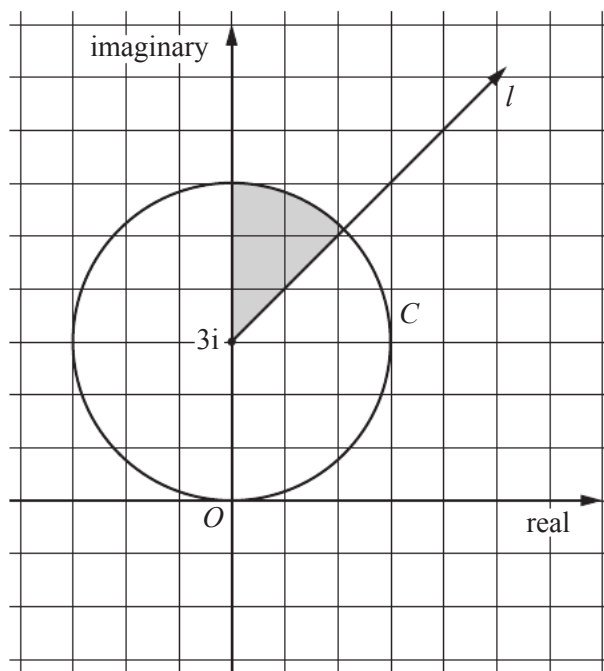
3 Use an algebraic method to find the square roots of  $11 + (12\sqrt{5})i$ . Give your answers in the form  $x + iy$ , where  $x$  and  $y$  are exact real numbers. [6]

4 The matrix  $\mathbf{M}$  is given by  $\mathbf{M} = \begin{pmatrix} 2 & 2 \\ 0 & 1 \end{pmatrix}$ . Prove by induction that, for  $n \geq 1$ ,

$$\mathbf{M}^n = \begin{pmatrix} 2^n & 2^{n+1} - 2 \\ 0 & 1 \end{pmatrix}. \quad [6]$$

5 Find  $\sum_{r=1}^n (4r^3 - 3r^2 + r)$ , giving your answer in a fully factorised form. [6]

6



The Argand diagram above shows a half-line  $l$  and a circle  $C$ . The circle has centre  $3i$  and passes through the origin.

(i) Write down, in complex number form, the equations of  $l$  and  $C$ . [4]

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] [3]

- 7 (i) Find the matrix that represents a rotation through  $90^\circ$  clockwise about the origin. [1]
- (ii) Find the matrix that represents a reflection in the  $x$ -axis. [2]
- (iii) Hence find the matrix that represents a rotation through  $90^\circ$  clockwise about the origin, followed by a reflection in the  $x$ -axis. [2]
- (iv) Describe a **single** transformation that is represented by your answer to part (iii). [2]

- 8 The cubic equation  $kx^3 + 6x^2 + x - 3 = 0$ , where  $k$  is a non-zero constant, has roots  $\alpha$ ,  $\beta$  and  $\gamma$ .  
Find the value of  $(\alpha + 1)(\beta + 1) + (\beta + 1)(\gamma + 1) + (\gamma + 1)(\alpha + 1)$  in terms of  $k$ . [6]

- 9 (i) Show that  $\frac{1}{3r-1} - \frac{1}{3r+2} \equiv \frac{3}{(3r-1)(3r+2)}$ . [2]

- (ii) Hence show that  $\sum_{r=1}^{2n} \frac{1}{(3r-1)(3r+2)} = \frac{n}{2(3n+1)}$ . [6]

- 10 The matrix  $\mathbf{A}$  is given by  $\mathbf{A} = \begin{pmatrix} a & 2 & 1 \\ 1 & 3 & 2 \\ 4 & 1 & 1 \end{pmatrix}$ .

- (i) Find the value of  $a$  for which  $\mathbf{A}$  is singular. [5]

- (ii) Given that  $\mathbf{A}$  is non-singular, find  $\mathbf{A}^{-1}$  and hence solve the equations

$$\begin{aligned} ax + 2y + z &= 1, \\ x + 3y + 2z &= 2, \\ 4x + y + z &= 3. \end{aligned}$$

[7]

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE



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